

Chapter 8: Decimals



DECIMALS

The numbers used to represent numbers smaller than unit 1 are called decimal numbers. The decimal point or the period plays a significant part in a Decimal Number. This period separates the fractional part and whole number part in a decimal number. Place value of a digit can be defined as the value of a digit as per the place of that digit in a number.

Decimals Examples

Let us consider a decimal number 0.5694 to see the different place values of each digit:

- The value of the digit at the first place after the decimal point is tenths place value. Tenths can be calculated as 1 unit divided in 10 equal parts i.e. 0.1. Therefore, considering this there are 5 tenths in the number 0.5694.
- The value of the digit at the second place after the decimal point is hundredths place value. The number in that place tells you how many hundredths are there. Hundredths can be calculated as 1 unit divided in100 equal parts i.e. 0.01. Therefore, considering this there are 6 hundredths in the number 0.5694
- The value of the digit at the third place after the decimal point is thousandths place value. The number in that place tells you how many thousandths are present. Thousandths can be calculated as 1 unit divided in1000 equal parts i.e. 0.001. Therefore, considering this there are 9 thousandths in the number 0.5694
- Ten-thousandths
- The value of the digit at the fourth place after the decimal point is ten-thousandths place value. The number in that place tells you how many ten-thousandths are present. Ten-thousandths can be calculated as 1 unit divided into 10000 equal parts i.e. 0.0001. Therefore, considering this there are 4 thousandths in the number 0.5694

Introduction to decimals

- Decimals are used in many ways in our lives.
- For example, in representing units of money, length and weight.
- Example: Price of the book is Rs. 34.5

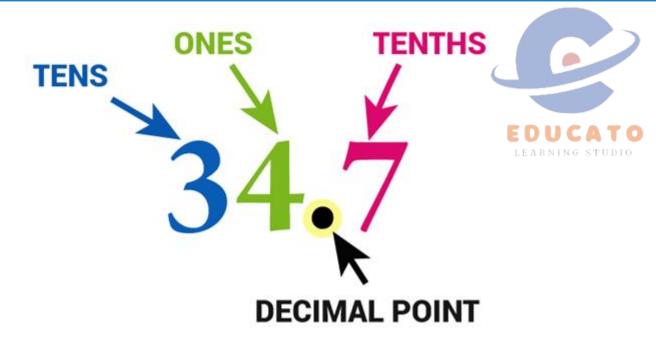
In Algebra, decimals are one of the types of numbers, which has a whole number and the fractional part separated by a decimal point. The dot present between the whole number and fractions part is called the decimal point. For example, 34.5 is a decimal number.

Here, 34 is a whole number part and 5 is the fractional part.

"." is the decimal point.

Let us discuss some other examples.

Here is the number "thirty-four and seven-tenths" written as a decimal number:



The decimal point goes between Ones and Tenths

34.7 has 3 Tens, 4 Ones and 7 Tenths

Types of Decimal Numbers

Decimal Numbers may be of different kinds, namely

Recurring Decimal Numbers (Repeating or Non-Terminating Decimals)

Example-

3.125125 (Finite)

3.121212121212..... (Infinite)

Non-Recurring Decimal Numbers (Non-Repeating or Terminating Decimals):

Decimal Fraction

It represents the fraction whose denominator in powers of ten.

Example:

81.75 = 8175/100

Converting the Decimal Number into Decimal Fraction:

For the decimal point place "1" in the denominator and remove the decimal point.

"1" is followed by a number of zeros equal to the number of digits following the decimal point.

For Example:

↓↓↓ 100

8 represents the power of 101 that is the tenths position.

1 represents the power of 100 that is the units position.

7 represents the power of 10-1 that is the one-tenths position.

5 represents the power of 10-2 that is the one-hundredths position.

So that is how each digit is represented by a particular power of 10 in the decimal number.

Place Value in Decimals

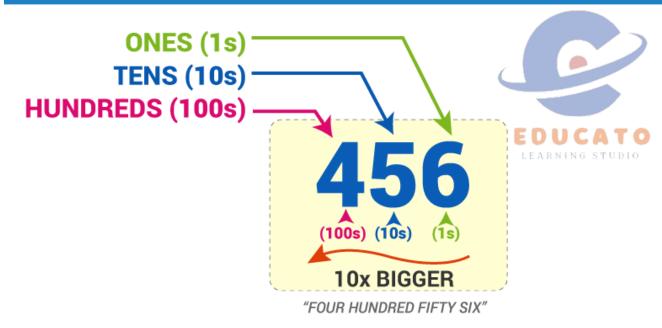
The place value system is used to define the position of a digit in a number which helps to determine its value. When we write specific numbers, the position of each digit is important.

Example:

For instance, let's consider the number 456.

- The position of "6" is in One's place, which means 6 ones (i.e. 6).
- The position of "5" is in the Ten's place, which means 5 tens (i.e. fifty).
- The position of "4" is in the Hundred's place, which means 4 hundred.
- As we go left, each position becomes ten times greater.

Hence, we read it as "Four hundred fifty-six".



As we move left, each position is 10 times bigger!

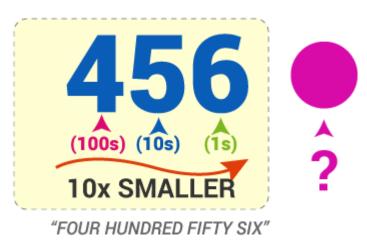
Tens are 10 times bigger than Ones.

Hundreds are 10 times bigger than Tens.

And

Each time we move right every position becomes 10 times smaller from Hundred's to Ten's, to Ones

But if we continue past Ones?

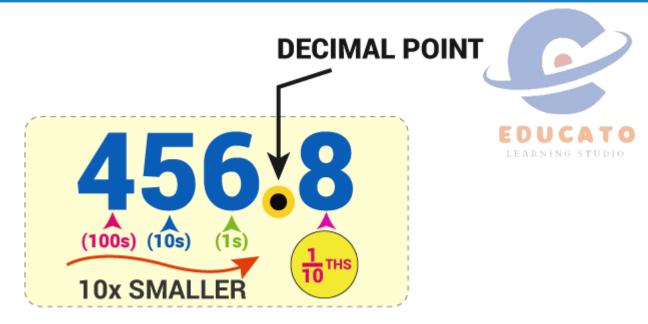


What are 10 times smaller than Ones?

 $\frac{1}{10}ths$ (Tenths) are!

Before that, we should first put a decimal point,

So we already know that where we put that decimal point.



We say the above example as four hundred and fifty-six and eight-tenths but we usually just say four hundred and fifty-six point eight.

The power of 10 can be found using the following Place Value Chart:

PLACE	VALU	E CHA	RT -									1 125		1
Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths	

The digits to the left of the decimal point are multiplied with the positive powers of ten in increasing order from right to left.

The digits to the right of the decimal point are multiplied with the negative powers of 10 in increasing order from left to right.

Following the same example 81.75

The decimal expansion of this is:

$$\{(8 \times 10) + (1 \times 1)\} + \{(7 \times 0.1) + (5 \times 0.01)\}$$

Where each number is multiplied by its associated power of ten.

Properties of Decimals

The important properties of decimal numbers under multiplication and division operations are as follows:

- If any two decimal numbers are multiplied in any order, the product remains the same.
- If a whole number and a decimal number are multiplied in any order, the product remains the same.
- If a decimal fraction is multiplied by 1, the product is the decimal fraction itself.
- If a decimal fraction is multiplied by 0, the product is zero (0).
- If a decimal number is divided by 1, the quotient is the decimal number.
- If a decimal number is divided by the same number, the quotient is 1.
- If 0 is divided by any decimal, the quotient is 0.
- The division of a decimal number by 0 is not possible, as the reciprocal of 0 does not exist.

Example:

3.2376 (Finite)

3.137654....(Infinite)

Representing decimals on a number line



Example: Represent 1.3 on the number line.

1.3 is more than 1 but less than 2.

Divide each unit length into 10 equal parts. From 1, take 3 parts to the right and we will get 1.3.

A Little Secret About Fractions and Decimals

Inter-conversion of fractions and decimals

Fractions as decimals

$$12/5 = 24/10 = (20+4) / 10 = 20/10 + 4/10 = 2+0.4 = 2.4$$

Decimals as fractions

$$21.2 = 21 + (2/10) = (210/10) + (2/10) = (212/10) = (106/5)$$

Recurring and terminating decimals

A terminating decimal is a decimal number which has finite number of digits after the decimal.

Example: 0.35

A recurring decimal is one where its decimal representation becomes periodic or same sequence of digits keeps repeating indefinitely.

Example: 31.213333...

Decimal 1 vs. Decimal 2

Comparing decimals

Any two decimal numbers can be compared by comparing their whole part and decimal parts. TO

If the whole parts are equal then the tenth parts can be compared and so on.

Comparison when whole parts equal

Compare 22.3 and 22.5

Whole parts are equal. Hence we compare tenths part.

$$22.5 = 22 + 5/10$$

Comparison whole parts are unequal

Compare 1 and 0.97

$$1 = 1 + \frac{0}{10} + \frac{0}{100}$$

$$0.97 = 0 + \frac{9}{10} + \frac{7}{100}$$

Whole part of 1 is greater than that of 0.97.

∴ 1 > 0.97

Tenths and Hundredths

 \Rightarrow 8 occupies the tenths position.

Move the tenths place to the right by one unit, the place value decreases further to 1/100th (hundredth) place.

⇒ 6 occupies the hundredths place

Place values

Place value gives a value of the number depending on its location.

Place value table of 5.2 is as shown below.

	Tens(10)	Ones(1)	$Tenths\left(\frac{1}{10}\right)$
5.2	0	5	2

Many Many Points Together

Addition of Decimals

Addition of 1.29 and 0.34

Ones	Tenths	Hundredths
1	2	9
0	3	4
1	6	3



Thus, 1.29 + 0.34 = 1.63

Subtraction of Decimals

Subtraction of 5.25 from 8.28

Ones	Tenths	Hundredths
8	2	8
-5	2	5
3	0	3

Thus, 8.28 - 5.25 = 3.03

Multiplication of Decimals

Multiplication of 2.8 and 7

 2.8×7 (There is only one digit to the right of the decimal point in 2.8)

 28×7 (Ignoring the decimals)

 $28 \times 7 = 196$

Now bring the decimal back after one digit from left and thus answer is 19.6

Division of Decimals

Divide 3.4 by 2

3.4/2 = Quotient

Ignore the decimal and divide the numerator by the denominator. Here, quotient = 34/2 = 17

Since there is only one digit to the right of the decimal, put the decimal after one digit from left in the quotient. Therefore, quotient becomes 1.7

Points on How to Use the Points

Point shift trick

Division: Point is shifted to the left by the number of zeroes in the denominator.

150/100=1.50 (Shifting decimal by 2 points to the left)

1.5/1000=0.0015 (Shifting decimal by 3 points to the left)

Decimals in length measurement

MATHEMATICS DECIMALS

1m = 100cm

1cm = 1/100m = 0.01m (Move decimal to left by 2 units in the numerator as there are two zeroes in the denominator)

150cm = 150/100m = 1.5m

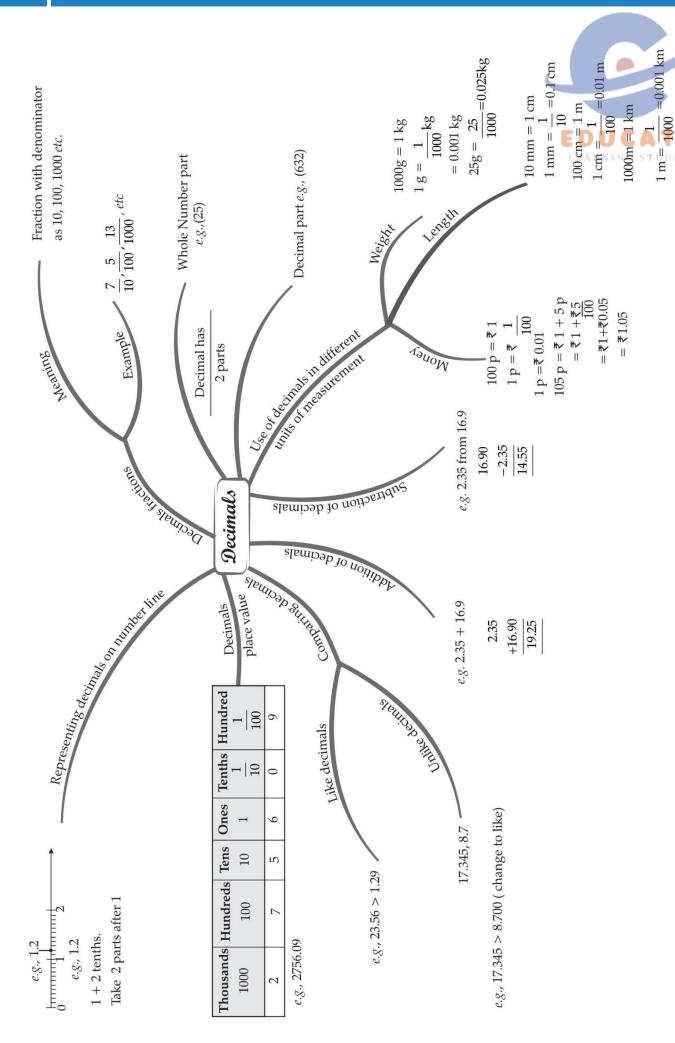
Decimals in weight measurement

1 kg = 1000 g

1 g = 1/1000 kg

298 g = 298/1000 kg = 0.298 kg (Move the decimal to the left by 3 places in the numerator as there are 3 zeroes in the denominator)





Important Questions

Multiple Choice Questions:

Question 1. 3 - tenths =

- (a) 0.3
- (b) 0.03
- (c) 0.003
- (d) 0.0003.

Question 2. Two tens and 2-tenths =

- (a) 20.2
- (b) 2.02
- (c) 202
- (d) none of these.

Question 3. One hundred and 1 - one =

- (a) 101
- (b) 1.01
- (c) 10.1
- (d) 0.101.

Question 4. Twelve point one =

- (a) 12.1
- (b) 12.01
- (c) 1.21
- (d) 0.121.

Question 5. $\frac{2}{10}$ =

- (a) 0.2
- (b) 0.02
- (c) 0.002
- (d) 0.0002

Question 6. $\frac{12}{10}$ =

- (a) 0.12
- (b) 1.2
- (c) 1.02



- (d) 1.002
- Question 7. $\frac{22}{10}$ =
- (a) 0.22
- (b) 2.2
- (c) 2.02
- (d) 2.002
- Question 8. 1 + $\frac{1}{10}$ =
- (a) 0.11
- (b) 1.1
- (c) 1.01
- (d) 1.001
- Question 9. $\frac{5}{2}$ =
- (a) 0.5
- (b) 0.2
- (c) 2.5
- (d) 0.25
- Question 10. $\frac{3}{5}$ =
- (a) 0.6
- (b) 0.006
- (c) 0.0006
- (d) 0.06
- Question 11. $2\frac{1}{10}$ =
- (a) 2.1
- (b) 2.01
- (c) 2.001
- (d) 2.0002
- Question 12. $\frac{16}{5}$ =
- (a) 0.32
- (b) 3.2
- (c) 3.02
- (d) 3.002



Question 13. 0.4 =

- (a) $\frac{1}{5}$
- (b) $\frac{2}{5}$
- (c) $\frac{3}{5}$
- (d) $\frac{4}{5}$

Question 14. 1.5 =

- (a) $\frac{1}{2}$
- (b) $\frac{5}{2}$
- (c) $\frac{3}{2}$
- (d) $\frac{7}{2}$

Question 15. 3.2 =

- (a) $\frac{16}{5}$
- (b) $\frac{8}{5}$
- (c) $\frac{32}{5}$
- (d) $\frac{24}{5}$

Match The Following:

	Column I		Column II	
1.	2m 5cm	A.	20 Rupees 15 paise	
2.	14.035 km	В.	Rs. 0.50	
3.	Rs 20.15	C.	2.05 m	
4.	+50 paise	D.	14km 35m	

Fill in the blanks:

- 25. 135 = 25 + $\frac{1}{}$ + $\frac{3}{}$ + $\frac{5}{}$ 1.
- $66.606 = 60 + \underline{} + \frac{6}{} + 0 + \frac{6}{}$ 2.
- $14 + \frac{5}{1000} + \frac{6}{10} = 14. _0_.$ 3.



4.
$$\frac{4}{100} + 5 =$$
_____. 0____

True /False:

- Rs. 60.75 can also expressed as 60Rs 75p. 1.
- 2. Adding 70.25 and 14.6 gives 84.85.
- 1 kg is equals to 100 grams 3.
- Expanded from of 23.05 is $20 + 3 + \frac{0}{10} + \frac{5}{100}$. 4.

Very Short Questions:

- Arrange the following decimal numbers in descending order. 8.51, 8.06, 9.10, 8.57
- Express $\frac{6}{100}$ as decimal. 2.
- 3. Express the following as cm using decimals: 30 mm
- 4. Show 1.1 on the number line.
- 5. Express the following as cm using decimals 116 mm.
- 6. Subtract 0.314 kg from 2.107 kg.
- 7. Subtract the sum of 114.753 and 70.14 from the sum of 93.12 and 212.15.
- 8. Write the following in decimals. 5 hundreds 3 tens 8 ones 4 tent
- 9. Write 14.3 in place value tab
- 10. Write the following in decimals
 - $(a) \frac{3}{4}$
- $(b) \frac{2}{5}$

Short Questions:

- 1. Represent 1.3,3.8 and 4.1 on the number line.
- 2. Write each of the following as decimals:

- (a) $\frac{8}{10}$ (b) $\frac{13}{100}$ (c) $\frac{256}{100}$ (d) $\frac{3}{1000}$
- 3. Convert the following unlike decimals into like decimals:

$$\frac{7}{10}, \frac{5}{50}, \frac{6}{5}, \frac{4}{20}$$

- **4.** Write the following decimals in their expanded form:
 - (a) 27.65
 - (b) 102.05



- (c) 36.36
- (d) 0.507
- **5.** Write each of the decimals as a mixed fraction.
 - (a) 95.8
 - (b) 15.78
 - (c) 0.015
 - (d) 19.91
- **6.** Express each of the following in terms of litres (L) using decimals:
 - (a) 625 mL
 - (b) 760 mL
 - (c) 11 L 125 mL
 - (d) 7 L 350 mL
- **7.** Write 3.03, 2.75 and 2.5 in ascending order.
- **8.** Find the value of the following:
 - (a) 15 9.363
 - (B) 5.28 1.4 + 3.116

Long Questions:

- Mr. Ranjan purchased 15.500 kg rice, 25.750 kg flour and 3.250 kg sugar. Find the 1. total weight of his purchases. Is it 50 kg or less? If less, how much less?
- 2. Ten years old Rahul can carry a maximum weight of 15 kg. If he bought 4 kg 900 g of apples, 2 kg 600 g of grapes and 5 kg 300 g of mangoes. Can he carry the total weight that he bought. If yes, then how much more weight he can carry with him?

Assertion and Reason Questions:

1.) Assertion (A) - 3 /tenths = 0.03

Reason (R) – Decimals are a set of numbers lying between integers on a number line

- a) Both A and R are true and R is the correct explanation of A
- b) Both A and R are true but R is not the correct explanation of A
- c) A is true but R is false
- d) A is false but R is true
- 2.) Assertion (A) Two tens and 2-tenths =100.2

Reason (R) – Decimals are a set of numbers lying between integers on a number line

- a) Both A and R are true and R is the correct explanation of A
- b) Both A and R are true but R is not the correct explanation of A



- c) A is true but R is false
- d) A is false but R is true

ANSWER KEY -



Multiple Choice questions:

- **1.** (a) 0.3
- **2.** (a) 20.2
- **3.** (a) 101
- **4.** (a) 12.1
- **5.** (a) 0.2
- **6.** (b) 1.2
- **7.** (b) 2.2
- **8.** (b) 1.1
- **9.** (c) 2.5
- **10.** (a) 0.6
- **11.** (a) 2.1
- **12.** (b) 3.2
- **13.** (b) $\frac{2}{5}$
- **14.** (c) $\frac{3}{2}$
- **15.** (a) $\frac{16}{5}$

Match The Following:

	Column I		Column II
1.	2m 5cm	C.	2.05 m
2.	14.035 km	D.	14km 35m
3.	Rs 20.15	A.	20 Rupees 15 paise
4.	4. +50 paise		Rs. 0.50

Fill in the blanks:

1. 25. 135 = 25 +
$$\frac{1}{10}$$
 + $\frac{3}{100}$ + $\frac{5}{1000}$

2.
$$66.606 = 60 + \underline{6} + \frac{6}{10} + 0 + \frac{6}{1000}$$

3.
$$14 + \frac{5}{1000} + \frac{6}{10} = 14.605$$
.

4.
$$\frac{4}{100}$$
 + 5 = **5** 0 **4**

True /False:

- **1.** True
- 2. True
- **3.** False
- 4. True

Very Short Answer:

1. The given decimal numbers are like decimals. By checking the digits before decimal, we observe that 9.10 is largest of given numbers. Among the remaining numbers, by checking the tenths place, we observe that 8.06 is the least. Out of remaining numbers, by checking the hundredths place, we observe that 8.57 is bigger than 8.51. So, by arranging the given numbers from big to small, we have,

$$\Rightarrow$$
 9.10, 8.57, 8.51, 8.06

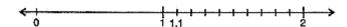
2.
$$\frac{6}{100} = 0.06$$

$$1 \text{ mm} = \frac{1}{10} \text{ cm}$$

3.
$$30 \text{ mm} = \frac{30}{10} \text{ cm}$$

$$3 \text{ cm} = 3.0 \text{ cm}$$

4. We know that 1.1 is more than one but less than two. There is one-tenth in it. Divide the unit length between 1 and 2 into 10 equal parts and take 1 part as shown below:



116 mm =
$$\frac{116}{10}$$
 cm [:::1 mm = $\frac{1}{10}$ = $\frac{1}{10}$ cm]
= $\frac{110+6}{10}$ cm
= $\left(\frac{110}{10} + \frac{6}{10}\right)$ cm = $\left(11 + \frac{6}{10}\right)$ cm
= 11.6 cm

6.



7.

Sum of 114.753 and 70.14:-
$$\frac{+\frac{114.753}{70.140}}{184.893}$$
 Sum of 212.15 and 93.12:- $\frac{+\frac{212.15}{93.12}}{305.27}$
Difference in their sums = $\frac{-\frac{305.270}{184.893}}{120.377}$

8. 5 Hundreds + 3 Tens + 8 Ones + 4 Tenths $= 5 \times 100 + 3 \times 10 + 8 \times 1 + 4 \times \frac{1}{10}$ $=500 + 30 + 8 + \frac{4}{10}$ $=538+\frac{4}{10}=538.4$

9.

Hundreds	Tens	Ones	Tenths
0	1	4	3

10.

(a)
$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$$

(b)
$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$$

Short Answer:

A BC 0 11.3 2 3 3.844.1 5 1.

A represents 1.3

B represents 3.8

and C represents 4

2.

(a)
$$\frac{8}{10} = 0.8$$
 (b) $\frac{13}{100} = 0.13$

(b)
$$\frac{13}{100} = 0.13$$

$$(c)\frac{256}{100} = 25.6$$

$$(c)\frac{256}{100} = 25.6$$
 $(d)\frac{3}{1000} = 0.003$



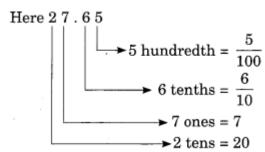
3. LCM of 10, 50, 5 and 20 = 100

$$\therefore \frac{7}{10} = \frac{7 \times 10}{10 \times 10} = \frac{70}{100}$$
$$\frac{5}{50} = \frac{5 \times 2}{50 \times 2} = \frac{10}{100}$$
$$\frac{6}{5} = \frac{6 \times 20}{5 \times 20} = \frac{120}{100}$$
$$\frac{4}{20} = \frac{4 \times 5}{20 \times 5} = \frac{20}{100}$$

So, the equivalent like decimals are

$$\frac{70}{100},\frac{10}{100},\frac{120}{100}$$
 and $\frac{20}{100}$ respectively.

4. (a) 27.65



Thus, expanded form of 27.65 = 20 + 7 +

$$\frac{6}{10} + \frac{5}{100}$$

(b) 102.05

Here, 1 0 2 . 0 5
$$\begin{array}{|c|c|c|c|c|c|}
\hline
& 5 \text{ hundredth} = \frac{5}{100} \\
& > 0 \text{ tenths} = \frac{0}{10} \\
& > 2 \text{ ones} = 2 \\
& > 0 \text{ tens} = 0 \\
& > 1 \text{ hundred} = 100
\end{array}$$

Thus, expanded form of $102.05 = 100 + 2 + \frac{5}{100}$

(c) 36.36

Here,
$$36.36$$

$$6 \text{ hundredth} = \frac{6}{100}$$

$$3 \text{ tenths} = \frac{3}{10}$$

$$6 \text{ ones} = 6$$

$$3 \text{ tens} = 30$$



Thus, expanded form of 36.36 = 30 + 6 + $\frac{3}{10} + \frac{6}{100}$

(d) 0.507

Here,
$$0.507$$

$$7 \text{ Thousandths} = \frac{7}{1000}$$

$$0 \text{ hundredth} = \frac{0}{100}$$

$$5 \text{ tenths} = \frac{5}{10}$$

$$0 \text{ ones} = 0$$

:. Expanded form of
$$0.507 = 0 + \frac{5}{10} + \frac{7}{1000}$$

5.

(a)
$$95.8 = 95 + 0.8 = 95 + \frac{8}{10} = 95 + \frac{4}{5} = 95\frac{4}{5}$$

Thus $95.8 = 95\frac{4}{5}$

(b)
$$15.78 = 15 + 0.78 = 15 + \frac{78}{100} = 15 + \frac{39}{50} = 15 \frac{39}{50}$$

Thus, $95.78 = 15 \frac{39}{50}$

(c)
$$0.015 = 0 + 0.015 = 0 + \frac{15}{1000} = \frac{15}{1000} = \frac{3}{200}$$

Thus, $0.015 = \frac{3}{200}$

(d)
$$19.91 = 19 + \frac{91}{100} = 19 \frac{91}{100}$$
Thus, $19.91 = 19 \frac{91}{100}$

6. (a) We know that 1000 mL = 1 L

∴ 625mL =
$$\frac{625}{1000}$$
 L = 0.625 L

Thus, 625 mL = 0.625 L

(b) We know that 1000 mL = 1 L

∴ 760 mL =
$$\frac{760}{1000}$$
 L

Thus,
$$760 \text{ mL} = 0.7$$

(c) We know that 1000 mL = 1 L

∴ 11L 125L = 11 L +
$$\frac{125}{1000}$$
 L

(d) We know that 1000 mL = 1 L

$$\therefore$$
 7 L 350 mL = 7 L + $\frac{350}{1000}$ L

7. The given decimals are unlike.

: Their corresponding like decimals are 3.03, 2.75 and 2.50.

Now neglecting the decimals, we have 303, 275 and 250.

∴ Ascending order is 2.50 < 2.75 < 3.03

8.

$$(a) \quad \begin{array}{r} 1 \stackrel{4}{\cancel{5}} \stackrel{9}{\cancel{0}} \stackrel{9}{\cancel{0}} \stackrel{10}{\cancel{0}} \\ - \quad 9 \stackrel{\cdot}{\cancel{0}} \stackrel{3}{\cancel{0}} \stackrel{6}{\cancel{0}} \stackrel{3}{\cancel{0}} \\ \hline \quad 5 \stackrel{\cdot}{\cancel{0}} \stackrel{6}{\cancel{0}} \stackrel{3}{\cancel{0}} \stackrel{7}{\cancel{0}} \end{array}$$

(b)
$$5.280$$
 and 8.396
 $+3.116$ -1.400
 8.396 6.996

Hence,
$$5.28 - 1.4 + 3.116 = 6.996$$

Long Answer:

1. Weight of rice = 15.500 kg

Weight of flour = 25.750 kg

Weight of sugar = 3.250 kg

Total weight of this purchases = 15.500 kg + 25.750 kg + 3.250 kg

= 44.500 kg

We see that the total weight of his purchases is less than 50 kg.





Thus, the total weight 44.500 kg is 5.500 kg less than 50 kg.

2. Weight of apples = 4 kg 900 g

Weight of grapes = 2 kg 600 g

Weight of mangoes = 5 kg 300 g

∴ Total weight of his purchases = 4 kg 900 g + 2 kg 600 g + 5 kg 300 g

$$= 4.900 \text{ kg} + 2.600 \text{ kg} + 5.300 \text{ kg}$$

$$= 12.800 \text{ kg}$$

But Rahul can carry a maximum weight of 15 kg.

Thus, more weight that he can carry with him = 15 kg - 12.800 kg

$$\begin{array}{r}
 \begin{array}{r}
 & 10 \\
 & \cancel{5} & \cancel{0} & 00 \\
 & -12.800 \\
 & 2.200
\end{array}$$

$$= 2.200 kg$$

Assertion and Reason Questions:

- 1) d) A is false but R is true
- 2) d) A is false but R is true