Mathematics

Chapter 5: Understanding Elementary Shapes

UNDERSTANDING ELEMENTARY SHAPES

Measuring Line Segments

A ruler or a divider is generally used to find the lengths of line segments. The length of a line segment can be defined as the distance between its endpoints.

Angles and its Types

An angle is made up of any two rays which have the same endpoint or starting point. It can be better understood by the movement of clock-hands. When a clock hand moves, it forms an angle. To measure the angle, a protractor is used. It should be noted that a straight angle is 180 degrees while a right angle is 90 degrees.

Based on the degree, an angle can be classified into 3 main types:

- **Acute angle:** When an angle measure is less than a right angle, it is called an acute angle.
- **Obtuse angle:** When an angle measure more than a right angle but less than a straight angle, it is called an obtuse angle.
- **Reflex angle:** When an angle measure more than a straight angle, it is called a reflex angle.

It should be noted that two intersecting lines are perpendicular if the angle formed between them is 90 degrees.

Triangles and its Types

A triangle is a closed figure having three sides and three interior angles. A triangle can be classified based on its length of sides and by its angles. The detailed classification of triangles is given below.

Based on	Triangle Name	Description
Based on its sides	Scalene triangle	All three sides are unequal.
	Isosceles triangle	Any two sides are equal.
	Equilateral triangle	All three sides are equal.

Based on its angles	Acute angled triangle	All the angles are acute.
	Right-angled triangle	Anyone angle is the right angle.
	Obtuse angled triangle	Anyone angle is obtuse.

Polygons

Polygons are closed geometric shapes having at least 3 sides and 3 angles. Based on the number of sides, a polygon can be categorized into multiple types. Some of the most common polygons are:

Polygon Name	No. of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

Quadrilaterals

As mentioned in the table above, a quadrilateral is one of the types of polygons having 4 sides and 4 angles. A quadrilateral can be categorized into 5 main types which are explained below.

Quadrilateral Type	Property
Rhombus	It has 4 sides of equal length.

Square	It is a rhombus with 4 right angles.
Parallelogram	It has two pairs of parallel sides.
Rectangle	It is a parallelogram of 4 right angles.
Trapezium	It has one pair of parallel sides.

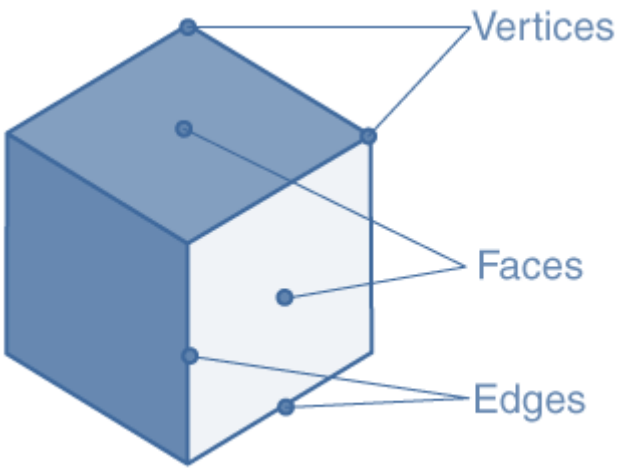
Solid Shapes or 3D Shapes

A solid shape or three-dimensional shape (3D shape) can be defined as the objects which can be measured in three directions i.e. length, breadth, and height. Examples of 3d shapes are cylinder, cube, cuboid, sphere, etc. Check out three-dimensional shapes to learn more about them and to get acquainted with the terms related to them.

Three Dimensional Shapes

Faces, Edges, and Vertices of Three-Dimensional Shapes

Three-dimensional shapes have many attributes, such as vertices, faces, and edges. The flat surfaces of the 3D shapes are called faces. The line segment where two faces meet is called an edge. A vertex is a point where three edges meet.



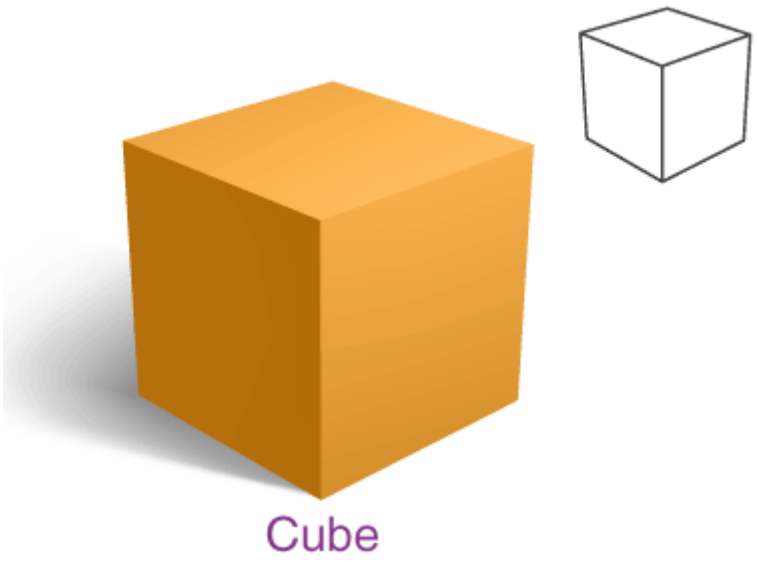
Three-dimensional Shapes Names:

- Cube
- Cuboid
- Cone
- Cylinder
- Sphere
- Pyramid
- Prism

Cube

A cube is a solid or three-dimensional shape which has 6 square faces. The cube has the following properties.

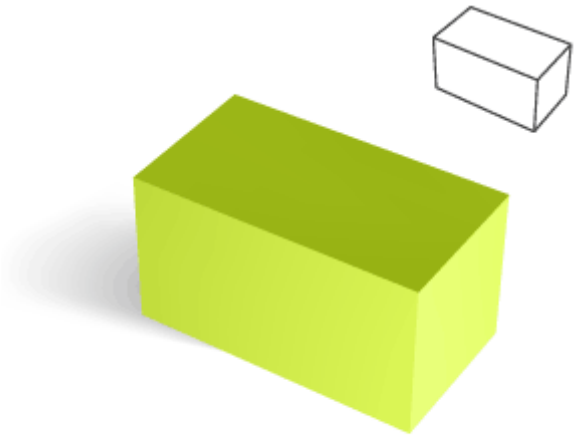
- All edges are equal
- 8 vertices
- 12 edges
- 6 faces



Cuboid

A cuboid is also called a rectangular prism, where the faces of the cuboid are a rectangle in shape. All the angles measure 90 degrees. The cuboid has

- 8 vertices
- 12 edges
- 6 faces

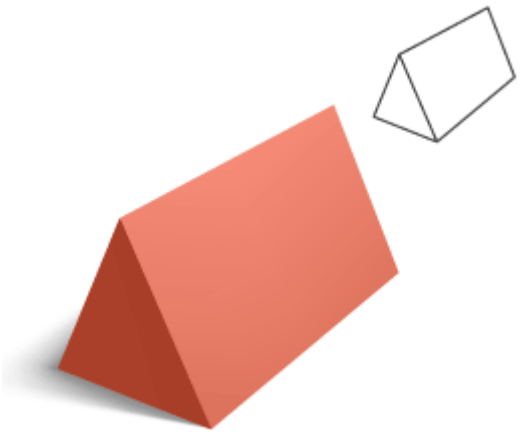


Cuboid

Prism

A prism is a 3D shape which consists of two equal ends, flat surfaces or faces, and also has identical cross-section across its length. Since the cross-section looks like a triangle, the prism is generally called a triangular prism. The prism does not have any curve. Also, a prism has

- 6 vertices
- 9 edges
- 5 faces – 2 triangles and 3 rectangles



Prism

Pyramid

A pyramid a solid shape, whose outer faces are triangular and meet to a single point on the top. The pyramid base can be of any shape such as triangular, square, quadrilateral or in the shape of any polygon. The most commonly used type of a pyramid is the square pyramid, i.e., it has a square base and four triangular faces. Consider a square pyramid, it has

- 5 vertices
- 8 edges
- 5 faces

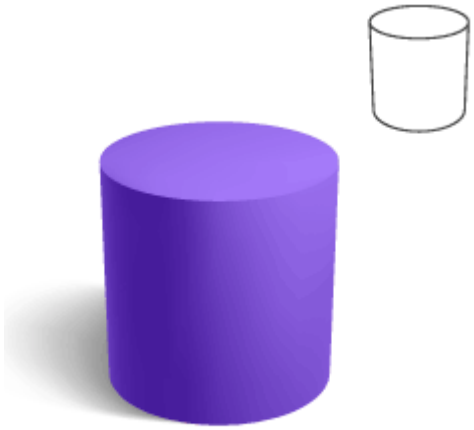
Pyramid

Cylinder

A cylinder is defined as a three-dimensional geometrical figure which has two circular bases connected by a curved surface. A cylinder has

- No vertex
- 2 edges

- 2 flat faces – circles
- 1 curved face

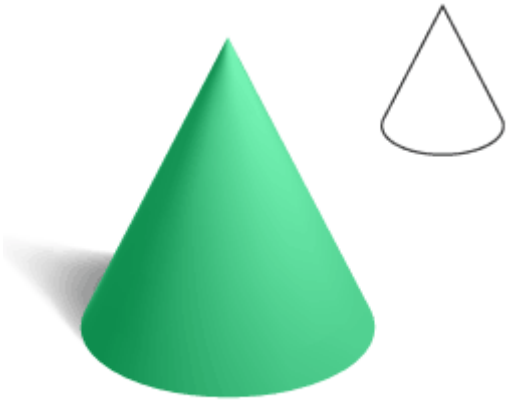


Cylinder

Cone

A cone is a three-dimensional object or solid, which has a circular base and has a single vertex. The cone is a geometrical figure that decreases smoothly from the circular flat base to the top point called the apex. A cone has

- 1 vertex
- 1 edge
- 1 flat face – circle
- 1 curved face

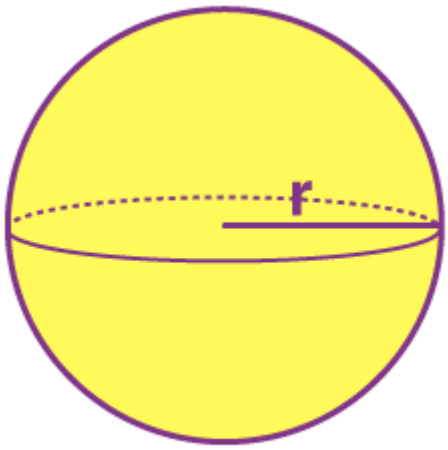


Cone

Sphere

A sphere is a three-dimensional solid figure which is perfectly round in shapes and every point on its surface is equidistant from the point is called the center. The fixed distance from the center of the sphere is called a radius of the sphere. A sphere has

- No vertex
- No edges
- 1 curved face



Sphere

Measuring line segments

The distance between the endpoints of a line segment is called its length.

⇒ Line segments can be measured by

- Comparison by observation
- Comparison by tracing
- Comparison using ruler and divider

Positioning error

To get the correct measure, the eye should be correctly positioned, just vertically above the mark. Errors can happen due to angular viewing.

Perpendicular Lines

Perpendicular Lines and perpendicular bisector

- When two lines intersect and the angle between them is a right angle, then the lines are said to be perpendicular.
- A perpendicular to a line segment that divides it exactly at the midpoint is called the perpendicular bisector.

Classification of Triangles

- Triangles are those closed figures which have exactly three sides.
- Based on their sides and angles, they can be classified into different triangles.

Types of Triangles based on lengths of sides

Based upon the length of the sides, triangles are classified as:

- Scalene
- Isosceles
- Equilateral

Types of Triangles based on angles

Based upon the measure of the angles, triangles are classified as:

- Acute-angled
- Obtuse-angled
- Right-angled

Quadrilaterals

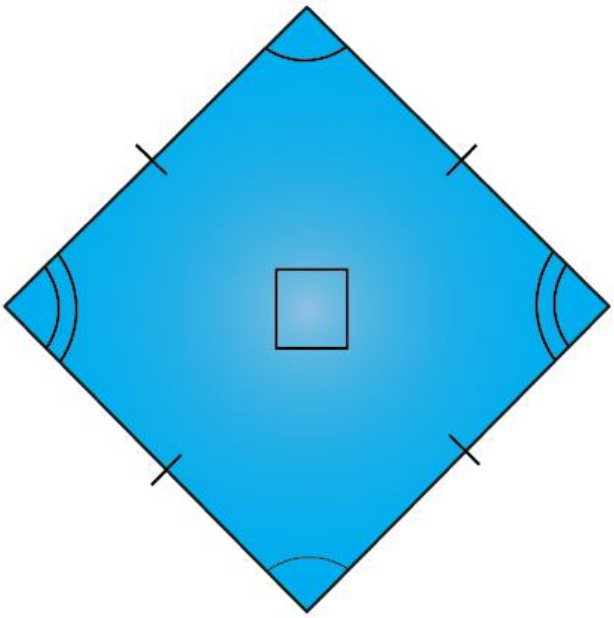
A quadrilateral is a polygon which has four sides.

Comparisons between different quadrilaterals

- Different quadrilaterals can be classified based on the lengths of the sides and angles.
- To know more about Quadrilateral, visit here.

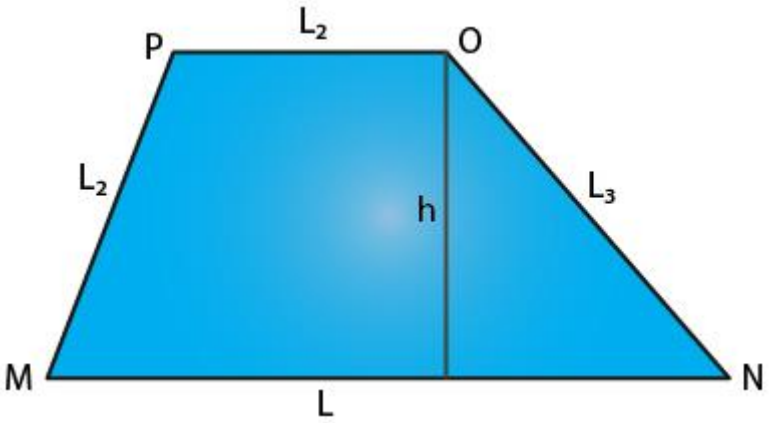
Rhombus

- A rhombus is a special type of parallelogram where all its sides are equal.
- The diagonals are perpendicular to each other. They also bisect the angles.



Trapezium

- A trapezium is a quadrilateral where only two sides are parallel to each other.
- No sides, angles and diagonals are congruent.



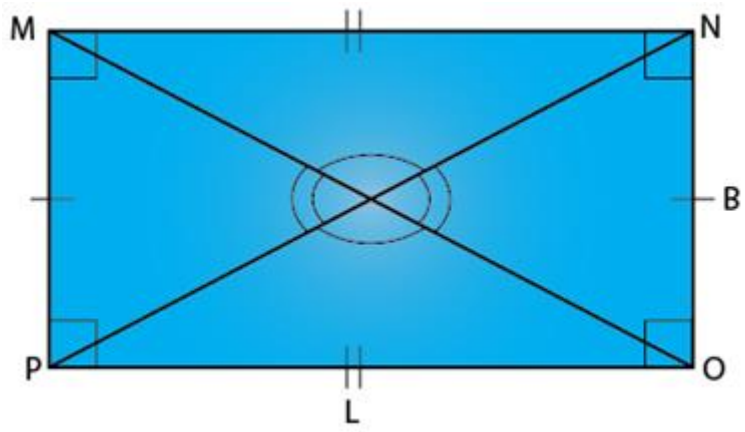
Polygons

- A polygon is a closed figure made up of line segments in two-dimension.
- Polygons are classified based on the number of sides.

Polygon	No. of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8

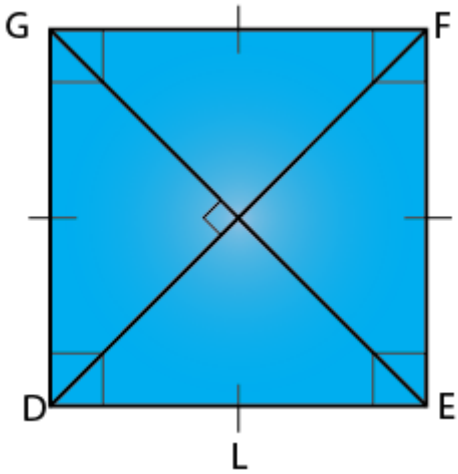
Rectangle

- A rectangle is a quadrilateral which has opposite sides equal and all angles are right angles.
- The diagonals are equal.



Square

- A square is a quadrilateral which has all sides equal and all angles are right angles.
- The diagonals are equal and perpendicular to each other.



Angles

The amount of rotation about the point of intersection of two planes (or lines) is called angle.

Right, straight and complete angles

- Right angle is equal to 900.
- Straight angle is equal to 1800.
- Complete angle is one complete revolution or equal to 360^0.

Acute, obtuse and reflex angles

- Acute angle is lesser than 900.
- Obtuse angle is greater than 900.
- Reflex angle is greater than 90^0.

Tools of construction

- Ruler and divider are used to measure lengths of line segments.
- A protractor is used to measure angles.

Measuring angles

- Angles are measured in degrees.
- Angles are measured by using a protractor.

MIND MAP : LEARNING MADE SIMPLE

CHAPTER-5

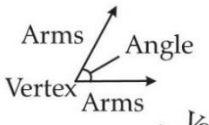
Understanding Elementary Shapes

Angles

Inclination between two straight lines

Arms — Rays making an angle

Vertex — common point



Types of angle

Angle	value
Acute	between 0 and 90°
Right	equal to 90°
Obtuse	between 90° and 180°
Straight	equal to 180°
Reflex	between 180 and 360°

Polygons

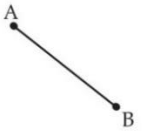
3 sides – triangle →
4 sides – quadrilateral →
5 sides – pentagon →
6 sides – hexagon →
8 sides – octagon →

Line segment

A line with two end points

Measurement of line segment

By observation — By just looking at them we can tell which one is longer



By tracing — To compare AB and CD we trace CD on a tracing paper and place it on AB and tell which one is longer

By using ruler and divider — Put one point of divider on A and open it till point B, now measure the stretched part on ruler

3D Shapes

Shapes having 3-Dimension length, breadth and height

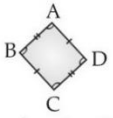
Quadrilateral

A plane figure bounded by 4 line segments

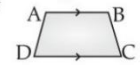
Rectangle:- A quadrilateral having equal opposite sides and each angle of 90°.

Square:- A quadrilateral having all 4 sides equal and each angle 90°.

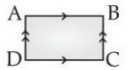
Rhombus:- A parallelogram having all 4 sides equal.



Trapezium:- A quadrilateral having only 1 pair of sides parallel.



Parallelogram:- A quadrilateral having opposite pair of sides parallel.



Triangle

3 sided figure having 3 angles and 3 vertices

Triangles based on sides

1. Scalene : triangle having all three sides unequal
2. Isosceles : triangle having 2 sides equal
3.Equilateral : triangle having all 3 sides equal

Triangles based on angles

1.Acute angled : Δ having all three angles acute (less than 90)
2.Obtuse angled : Δ having one of its angle obtuse (more than 90)
3.Right angled:Δ having one of its angle equal to 90°

3-D Shapes	Faces	Vertices	Edges	
Sphere	1	0	0	
Cone	2	1	1	
Cylinder	3	0	2	
Cuboid	6	8	12	
Cube	6	8	12	
Pyramid (Triangular)	4	4	6	

Important Questions

Multiple Choice Questions:

1. A quadrilateral having one pair of sides parallel is called:

   A. square

   B. trapezium

   C. rectangle

   D. None of these

2. A triangular prism has:

   A. 9 faces

   B. 8 faces

   C. 7 faces

   D. 5 faces

3. Where will the hand of a clock stop if it starts at 2 and makes of a revolution, clockwise?

   A. 5

   B. 8

   C. 11

   D. None of these

4. An angle whose measure is equal to half of a revolution is:

   A. right angle

   B. acute angle

   C. straight angle

   D. obtuse angle

5. A quadrilateral whose opposite sides are parallel is called:

   A. square

   B. rectangle

C. parallelogram

D. None of these

6. A quadrilateral whose all the sides are equal and each angle is 90° is called a:

A. square

B. rhombus

C. rectangle

D. trapezium

7. Where will the hand of a clock stop if it starts at 12 and makes of a revolution, clockwise?

A. 6

B. 9

C. 3

D. None of these

8. When the sum of the measures of two angles is that of a right angle, then each one of them is _____.

A. obtuse angle

B. acute angle

C. straight angle

D. right angle

9. How many degrees are there in two right angles?

A. 90°

B. 180°

C. 270°

D. 360°

10. An angle formed by two opposite rays is called a:

A. complete angle

B. zero angle

C. straight angle

D. right angle

11. Where will the hand of a clock stop if it starts at 3 and makes of a revolution, clockwise?

A. 6

B. 12

C. 9

D. None of these

12. How many centimeters make 3m?

A. 100

B. 30

C. 300

D. 3000

13. When an arm of an angle is extended then how does its measure change?

A. Doubled

B. Tripled

C. Remains the same

D. Halved

14. Triangle having the angles 40°, 30°, 110° is called:

A. acute angled triangle

B. obtuse angled triangle

C. right triangle

D. none of these

15. An angle which is greater than a right angle but less than a straight angle is called:

A. an acute angle

B. an obtuse angle

C. a complete angle

D. straight angle

Match The Following:

	Column I		Column II
1.	Cube	**A.**	6
2.	Square pyramid	**B.**	12
3.	Triangular prism	**C.**	8
4.	Triangular pyramid	**D.**	9

Fill in the blanks:

1. Measure of a complete angle is _____°.

2. The triangle in which _____ sides are equal is called isosceles triangle.

3. Each of its angles rectangle measures _____°.

4. A cube has _____ vertices.

True /False:

1. Sum of any two sides of a triangle is greater than the third side.

2. An equilateral triangle is also considered as an isosceles triangle

3. A polygon is regular if its all sides are equal.

4. Opposite faces of a cuboid are equal in size.

Very Short Questions:

1. How many faces a tetrahedron have?

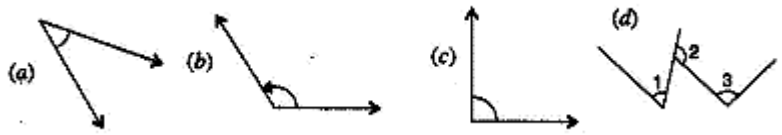
2. What is the angle name for half a revolution?

3. Draw a hexagon and write its sides and diagonals?

4. If B is the mid point of $(\overline{AC})$ and C is the point of $(\overline{BD})$ where A, B, C, D lie on a straight line, say why AB = CD?

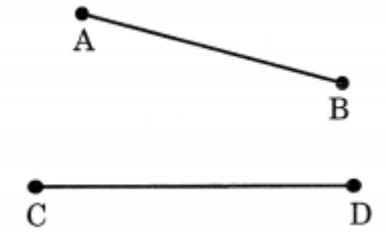
5. Draw a rough sketch of a regular octagon. Draw a rectangle by joining exactly four of the vertices of the octagon.

6. Measure the angles given below, using the Protractor and write down the measure.

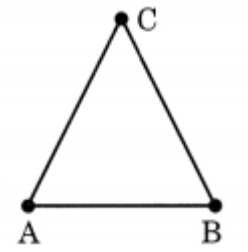


7. All equilateral triangle are isosceles, but all isosceles triangle are not equilateral. Justify the statement.

8. Which of the following line-segments is longer?



9. How many line segments are used in making a triangle?



10. What is the measure of straight angle?

Short Questions:

1. In the given figure, name the following angles as acute, obtuse, right, straight or reflex.

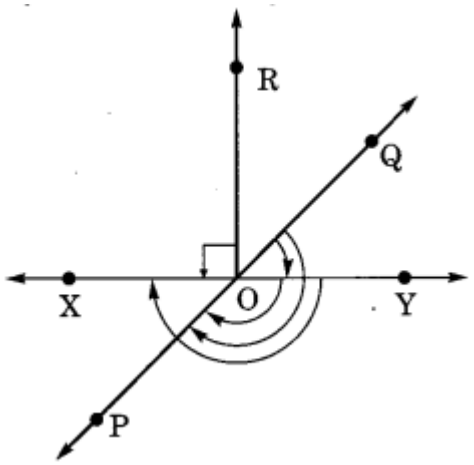
   (a) ∠QOY

   (b) ∠YOP

   (c) ∠ROX

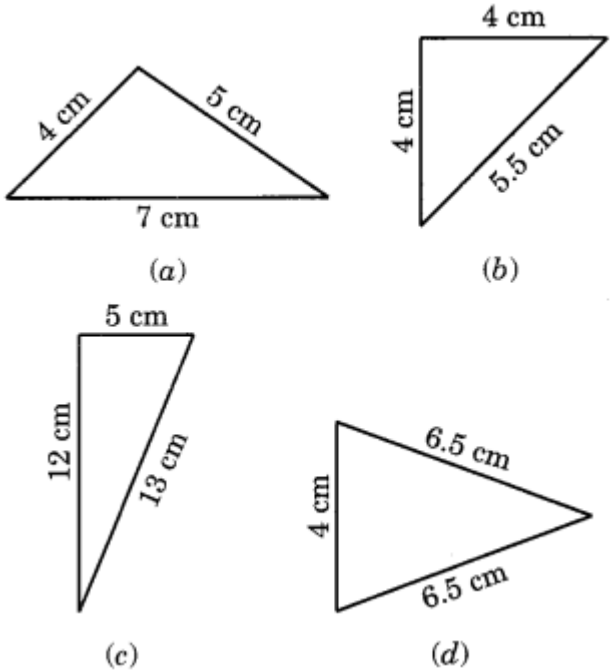
   (d) ∠QOX

   (e) ∠POQ

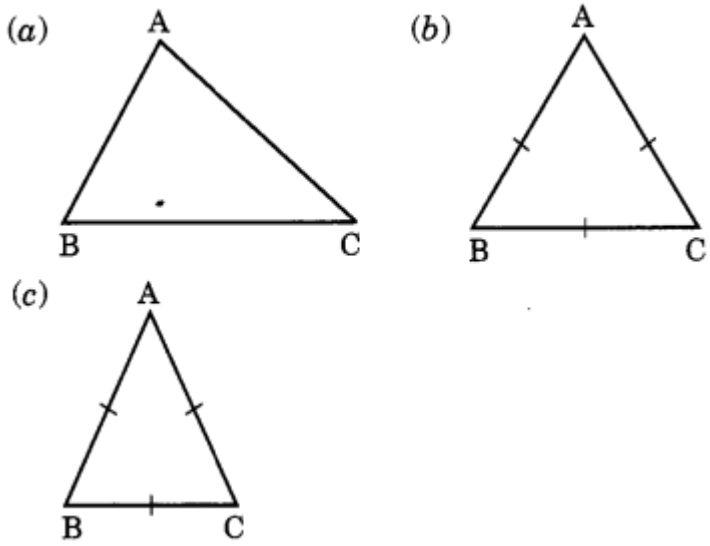


2. In the given figure, find the measure of the angles marked with a, b, c, d, e and f.

3. Classify the given triangles whose sides are indicated on them.



4. What are the types of the following triangles on the basis of sides?



Long Questions:

1. Verify the 'Euler's formula' V + F = E + 2 for the given figures.

(a) A triangular prism having 5 faces, 9 edges and 6 vertices.

(b) A rectangular prism with 6 faces, 12 edges and 8 vertices.

(c) A pentagonal prism with 7 faces, 15 edges and 10 vertices.

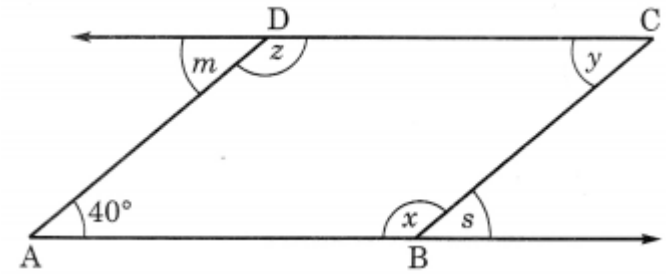
(d) A tetrahedron -with 4 faces, 6 edges and 4 vertices.

2. Complete the given table for prisms:

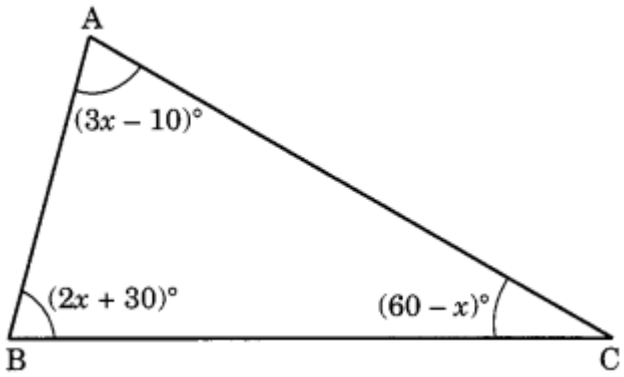
Prism	Number Of faces	Number Of edges	Number Of vertices
Triangular	–	–	–
Quadrilateral	–	–	–
Pentagonal	–	–	–

Hexagonal	–	–	–
Octagonal	–	–	–
Decagonal	–	–	–

3. In the given figure, find the values of x, y, z, s and m.



4. Find the value of x from the given figure and hence find the measure of each angle of the triangle.



Assertion and Reason Questions:

1.) Assertion (A) –A line segment is a fixed portion of a line

Reason (R) – We use this idea to compare line segments.

a) Both A and R are true and R is the correct explanation of A

b) Both A and R are true but R is not the correct explanation of A

c) A is true but R is false

d) A is false but R is true

2.) Assertion (A) – A triangle is made of three, a quadrilateral of four segments.

Reason (R) – The measure of each line segment is a unique number called its length.

a) Both A and R are true and R is the correct explanation of A

b) Both A and R are true but R is not the correct explanation of A

c) A is true but R is false

d) A is false but R is true

ANSWER KEY -

Multiple Choice questions:

1. **B.** trapezium
2. **D.** 5 faces
3. **B.** 8
4. **C.** straight angle
5. **C.** parallelogram
6. **A.** square
7. **B.** 9
8. **B.** acute angle
9. **B.** 180°
10. **C.** straight angle
11. **B.** 12
12. **C.** 300
13. **C.** Remains the same
14. **B.** obtuse angled triangle
15. **B.** an obtuse angle

Match The Following:

	Column I		Column II
1.	Cube	**B.**	12
2.	Square pyramid	**C.**	8
3.	Triangular prism	**D.**	9
4.	Triangular pyramid	**A.**	6

Fill in the blanks:

1. Measure of a complete angle is **360**°.
2. The triangle in which **two** sides are equal is called isosceles triangle.

3. Each of its angles rectangle measures **90º**.

4. A cube has **8** vertices.

True /False:

1. True

2. False. in isosceles triangle only two sides are equal.

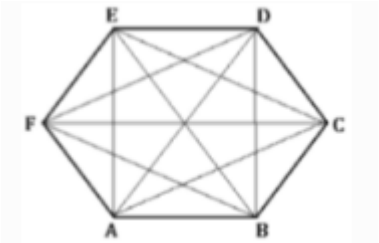
3. False. For a polygon to be regular, all sides as well as all angles have to be equal.

4. True.

Very Short Answer:

1. In geometry, a tetrahedron is a polyhedron composed of four triangular faces, three of which meet at each corner or vertex.
2. Straight Angle (180°)

3. Hexagon



Sides of hexagon: AB, BC, CD, DE, EF and FA.

Diagonals of hexagon: AC, AD, AE, BD, BE, BF, CE, CF, and DF

4.

∵ B is the mid-point of $\overline{AC}$

∴ AB = BC …(1)

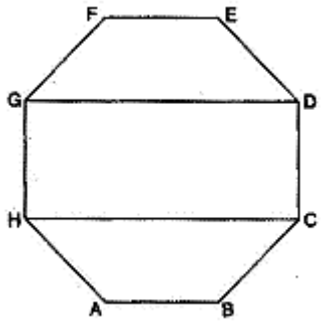
∵C is the mid-point of $\overline{BD}$

∴ BC = CD …(2)

In view of (1) and (2), we get

AB = CD.

5.



6. (a) 45°

(b) 125°

(c) 90°

(d) ∠1 = 40°, ∠2 = 125° and ∠3 = 95°.

7. An isosceles triangle is any triangle with 2 sides that are equal in length. So, every equilateral triangle is a special case of an isosceles triangle since not just 2 sides are equal, but all 3 are. But every isosceles triangle is not equilateral, because you can have 2 sides of equal length and a third side that is either longer or shorter than those 2 sides. For example, if the triangle is a right-angle triangle and the two sides that meet to make the right angle are the same length, then the 3rd side would be longer than those two.

8. By using divider, $\overline{CD}$ seems to be longer than $\overline{AB}$.

9. Three line segments are used to make a triangle.

10. The measure of straight angle is 180°.

Short Answer:

1. (a) ∠QOY = acute angle.

(b) ∠YOP = obtuse angle.

(c) ∠ROX = right angle.

(d) ∠QOX = reflex angle.

(e) ∠POQ = straight angle.

2. ∠a = 180° -129° = 51°

∠b = 180° – (51° + 92°)

= 180° – 143° = 37°

∠c = 180° – 88° = 92°

∠d = 180° – 152° = 28°

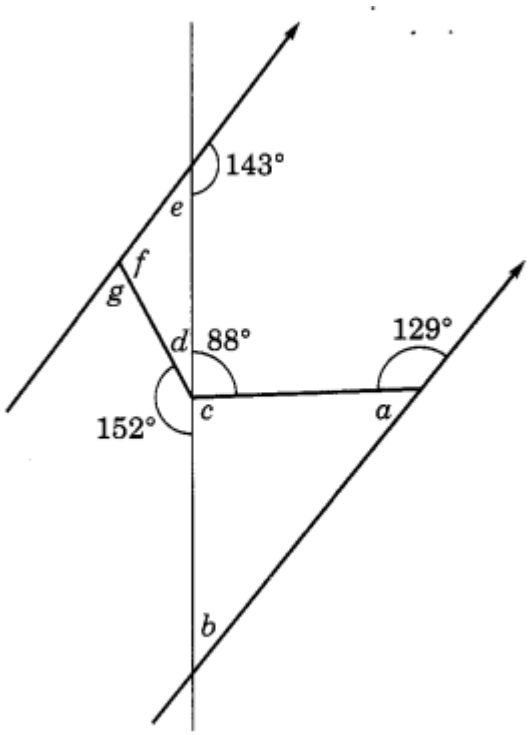
∠e = 180° – 143° = 37°

∠f= 180° – (∠e + ∠d)

= 180° – (37° + 28°)

= 180°- 65°= 115°

∠g = 180° – ∠f = 180° – 115° = 65°



3. (a) All sides are different. So, it is a scalene triangle.

(a) Lengths of two sides of the triangle are same. So, it is an isosceles triangle.

(b) All sides are unequal and one angle is right angle. So it is scalene right angled triangle.

(c) Two sides of this triangle are equal. So, it is an isosceles triangle.

4. (a) Scalene triangle.

(b) Equilateral triangle.

(c) Isosceles triangle

Long Answer:

1. (a) Here, F = 5, E = 9 and V = 6

∴ V + F = E + 2

⇒ 6 + 5 = 9 + 2

⇒ 11 = 11

Hence, verified.

(b) Here, F = 6, E = 12 and V = 8

∴ V + F = E + 2

⇒ 8 + 6 = 12 + 2

⇒ 14 = 14

Hence, verified.

(c) Here, F = 7, E = 15 and V = 10

∴ V + F = E + 2

⇒ 10 + 7 = 15 + 2

⇒ 17 = 17

Hence, verified.

(d) Here, F = 4, E = 6 and V = 4

∴ V + F = E + 2

⇒ 4 + 4 = 6 + 2

⇒ 8 = 8

Hence, verified.

2.

Prism	Number Of faces	Number Of edges	Number Of vertices
Triangular	5	9	6

Quadrilateral	6	12	8
Pentagonal	7	15	10
Hexagonal	8	18	12
Octagonal	10	24	16
Decagonal	12	30	20

3. Given that $\angle A = 40°$

(i) $\angle DAB + \angle ABC = 180°$ (adjacent angles)

$\Rightarrow 40° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 40° = 140°$

Hence, $\angle x = 140°$

(ii) $\angle x + \angle y = 180°$ (adjacent angles)

$\Rightarrow 140° + \angle y = 180°$

$\Rightarrow \angle y = 180° - 140° = 40°$

Hence, $\angle y = 40°$

(iii) $\angle y + \angle z - 180°$ (adjacent angles)

$\Rightarrow 40° + \angle z = 180°$

$\Rightarrow \angle z = 180° - 40° = 140°$

Hence, $\angle z = 140°$

(iv) $\angle x + \angle s = 180°$ (straight angles)

$\Rightarrow 140° + \angle s = 180°$

$\Rightarrow \angle s = 180° - 140° = 40°$

Hence, $\angle s = 40°$

(v) $\angle m + \angle z = 180°$ (straight angles)

$\Rightarrow \angle m + 140° = 180°$

$\Rightarrow \angle m = 180° - 140° = 40°$

4. (i) Sum of the three angles of a triangle = 180°

$\therefore 2x + 30° + 60° - x + 3x - 10° = 180°$

$\Rightarrow (2x - x + 3x) + (30° + 60° - 10°) = 180°$

$\Rightarrow 4x + 80° = 180°$

$\Rightarrow 4x = 180° - 80°$

$\Rightarrow 4x = 100°$

$\therefore x = 10004 = 25°$

$\therefore$ Measure of the angles are:

(i) $(2x + 30)° - 2 \times 25° + 30° = 80°$

(ii) $(60 - x)° = 60° - 25° = 35°$

(iii) $(3x - 10)° = 3 \times 25° - 10° = 75° - 10° = 65°$

Hence, $x = 25°$ and the angles of the triangles are: 80°, 35° and 65°.

Assertion and Reason Answers:

1) a) Both A and R are true and R is the correct explanation of A

2) b) Both A and R are true but R is not the correct explanation of A